Math 10

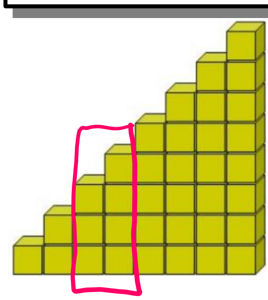
Section 7.0.2 Arithmetic Sequences

Focus:

1. To be able to derive a rule for determining the general term of an arithmetic sequence
2. To be able to determine t_n, a, d, or n in a problem that involves an arithmetic sequence
3. To be able to describe the relationship between an arithmetic sequence and a linear function
4. To be able to solve problems involving arithmetic sequences.

Curricular Competencies:
C2: I can represent math concretely, pictorially and symbolically

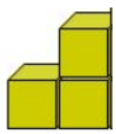
Staircase Numbers:



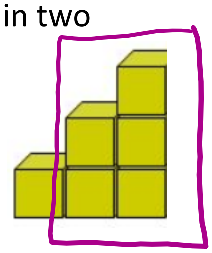
A staircase number is the number of cubes needed to make a staircase that has at least two steps. Is there a pattern to the number of cubes in successive staircase numbers? How could you predict different staircase numbers?

To generate a two-step staircase number, add the numbers of cubes in two consecutive columns.

The first staircase …



The second staircase …



Complete the table for the number of cubes required for each staircase number of a two-step staircase.

Term	Staircase Number (# of cubes needed)
1	3 (1+2)
2	5]2 (2+3)
3	7 (3+4)
4	9
5	11
6	13]2
7	15
8	17
9	19
10	21

Does the table of values represent a linear or non-linear relation?

increase by a constant of 2

Are the data discrete or continuous?

half a step could be dangerous

What is the pattern for the number of cubes in a two-step staircase?

staircase # = 2t + 1

How could you find the number of cubes in the 12th term? The 100th term?

$\# = 2(12)+1$
$= 25$

$\# = 2(100)+1$
$= 201$

Sequences

A sequence is an ordered list of objects that follow a pattern or rule to determine the next term in the sequence. Sequences can be finite (a limited number of terms) or infinite (unlimited number of terms).

An arithmetic sequence is an ordered list of terms in which the difference between consecutive terms is constant. This constant is called the common difference. If you subtract the first term from the second term for any two consecutive terms in the sequence, you will get the common difference. For example: 10, 16, 22, 28 … the common difference is 6. The general term of an arithmetic sequence is its rule.

$$t_n = a + (n-1)d$$

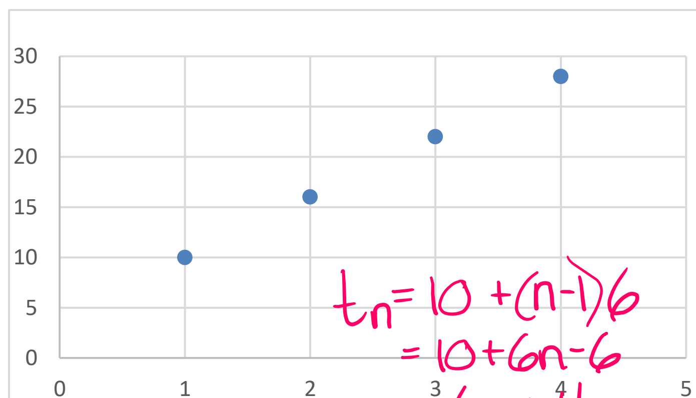
Where t_n= general term .... can be any term

a= first term (t_1)

n= number of terms

d= common difference

An arithmetic sequence is a linear relation. When graphed, an arithmetic sequence is a discrete linear relation with a constant rate of change equal to the common difference. The general term of the graph is $t_n = 10 + (n-1)d$. It can also be written as $t_n = 6n+4$ or with function notation $f(n) = 6n+4$.



$t_n = 10 + (n-1)6$
$= 10 + 6n - 6$
$= 6n + 4$

Examples

$t_n = a + (n-1)d$

For the arithmetic sequence 2, 9, 16, … determine

a. t_6 $t_6 = 2 + (6-1)7$
$= 37$

b. t_{45} $t_{45} = 2 + (45-1)7$
$= 310$

c. t_n $t_n = 2 + (n-1)7$
$= 2 + 7n - 7$
$= 7n - 5$

Given the arithmetic sequence 3, 10, 17, 24, … if one term in the sequence is 129, which term is it?

$t_n = a + (n-1)d$

$n = 19$

$129 = 3 + (n-1)7$

$129 = 3 + 7n - 7$

$129 = 7n - 4$

$133 = 7n$

Determine the values of a and d. State the missing terms of the sequence.

?, ?, ?, 17, 22

2 7 12 $\underbrace{\quad}_{5}$

$d = 5$

$a = 2$

?, 8, ?, ?, 17

$\boxed{a = 5}$

pretend...

$a = 8$

$t_4 = 17$

$t_n = a + (n-1)d$

$17 = 8 + (4-1)d$

$17 = 8 + 3d$

$9 = 3d$

$\boxed{3 = d}$

5, 8, 11, 14, 17

The muskox and the caribou of northern Canada are hoofed mammals that survived the Pleistocene Era, which ended 10 000 years ago. In 1972, the Banks Island muskox population was approximately 3800 animals. Suppose that in subsequent years, the growth of the muskox population generated an arithmetic sequence, in which the number of muskox increased by approximately 1650 each year. How many years would it take for the muskox population to reach 83 000?

$t_n = a + (n-1)d$

$83\,000 = 3800 + (n-1)1650$

$83\,000 = 3800 + 1650n - 1650$

$83\,000 = 2150 + 1650n$

$80\,850 = 1650n$

$49 = n$

49 years

A furnace technician charges \$65 for making a house call, plus \$42 per hour or portion of an hour.

a. Generate the possible charges (excluding parts) for the first 4 h of time.
b. Determine the general term, t_n, for the sequence. How do the values in the general term related to the situation?
c. What is the charge for 10 h of time?

$t_n = a + (n-1)d$

$= 65 + (4-1)42$

$= \$191$

$t_n = 65 + (n-1)42$

$= 65 + 42n - 42$

$= 42n + 23$

#of hours

set cost

per hour charge

$t_{10} = 65 + (10-1)42$

$= \$443$

Assign: worksheet